Synthesis of Amplitude Response of Optical Directional Coupler Modulators

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Abstract

Optical directional coupler modulator amplitude response may be synthesized using the Gel'fand-Levitan-Marchenko inverse scattering technique. The technique determines the coupling function required to obtain the specified response, and for this summary we illustrate it with the response function of a third order Butterworth function. The technique provides the possibility of designing coupler modulators with small switching voltages, and better linearity.

Summary

In order to avoid spectral broadening in optical communication systems, external modulators are used instead of direct modulation of the source. The modulators currently available today have nonlinear response functions, and the common Mach-Zehnder modulator [1] for instance has a squared cosine response. In order to avoid crosstalk in analog optical systems, the modulation depth then has to be extremely small. The optical directional coupler modulator with constant coupling also has a nonlinear response, and the response function has the form [1]:

$$\eta = \frac{1}{1+(\delta/\kappa)^2}\sin^2(\kappa z\left[1+(\delta/\kappa)^2\right]^{1/2}) \qquad (1)$$

where $\delta = \Delta\beta/2$, with $\Delta\beta$ the difference in the propagation constants of the two coupled optical modes, and κ is the coupling factor and η is proportional to the optical power in the output signal. However, it is possible to construct a directional coupler modulator with a response that differs dramatically from that of a uniform directional coupler, by varying the coupling function by means of a synthesis technique to obtain a given response function. In this paper we discuss how the coupling function may be synthesized from a specified response function using the Gel'fand-Levitan-Marchenko technique inverse scattering formulation. We believe that this the first time the synthesis of the amplitude response of optical coupler modulators has been proposed. We briefly outline the technique based on the coupled mode theory, with the usual notation, for the coupler modulator. The assumption made is that the coupling function varies slowly, so that its spatial derivative may be ignored, in the current analysis. A schematic diagram of a directional coupler with uniform coupling is given in figure 1.

The coupled mode equations of the directional coupler in general have no analytical solutions, since they lead to the Riccati equation [1], and therefore there are no general inverse technique from which we may find the coupling function from the response. However, if the response function is restricted to certain classes of polynomial functions it is possible to apply inverse techniques. The Gel'fand-Levitan-Marchenko (GLM) [2] technique may be used to find a coupling function that gives a result close to the specified one, using various approximation functions including of the Butterworth polynomial form, the Chebycheff polynomials or other polynomial approximations, of the specified response. Another strategy for finding an inverse solution is keep the original response function, but use approximation methods to find the coupling

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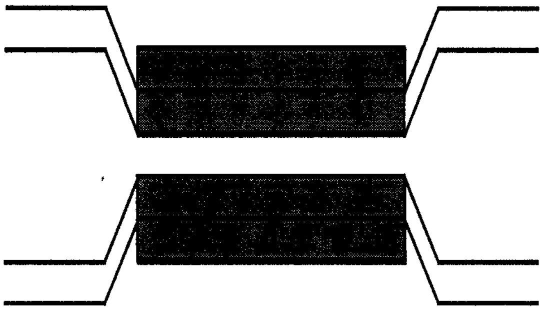

Figure 1: A schematic diagram of a directional coupler with uniform coupling

function, which we call the Fourier transform method, however, space does not permit the description of this method in this summary. We provide a short description of the GLM method following the notation of Song et al, and Winick [5],[3], [4] who have previously applied this technique to optical corrugated coupler filters . The first step is find a rational polynomial function $r(\delta)$ that best fits the required response function. For the coupler, in coupled mode theory, the complex amplitudes of the waves in the two guides are represented by $S(z,\delta)$ and $R(z,\delta)$, and r is the ratio $S(L,\delta)/R(L,\delta)$. The order of the denominator of r needs to be higher than the order of the numerator, and all the poles of the function need to be in the lower half plane and distinct. In addition, r must have the following property:

$$r^*(\delta^*) = -r(-\delta) \tag{2}$$

as otherwise κ will not necessarily be real, and $*$ superscript means complex conjugate. After an appropriate r is obtained, the following definitions are made:

$$\begin{aligned} u &= -j\delta && (3) \\ G(u) &= r(ju) = \frac{P(u)}{Q(u)} && (4) \\ F(u) &= Q(u)Q^*(-u^*) + P(u)P^*(-u^*) && (5) \end{aligned}$$

The next step is to find all the roots of $Q = 0$, which now will be in the left half plane, and denote them $\rho_1, \rho_2, \ldots\ldots \rho_N$. In addition the roots of $F = 0$ need to be found, and they can be denoted $\omega_1, \omega_2, \ldots, \omega_N, -\omega_1^*, -\omega_2^*, \ldots, -\omega_N^*$. N is here the order of the polynomial Q. After all these roots are found, the next step in the method is to solve the following set of $2N$ coupled linear equations:

$$\sum_{m=1}^{N} \left\{ \frac{e^{-\omega_m z}}{G(\omega_m)(\rho_n - \omega_m)} \begin{bmatrix} g_{1,m}(z) \\ g_{2,m}(z) \end{bmatrix} - \frac{e^{\omega_m^* z}}{\rho_n + \omega_m^*} \begin{bmatrix} g_{2,m}^*(z) \\ -g_{1,m}^*(z) \end{bmatrix} \right\} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{6}$$

Here the index n runs from $1 \ldots N$. Note that there are in fact $4N$ unknowns in the equation set, as the real and imaginary parts have to be found separately. Once all the functions $g_{i,j}$ are found, the coupling function may now be calculated from:

$$\kappa(z) = -2j \sum_{n=1}^{N} \left[g_{1,n}(z) e^{\omega_n z} - G(-\omega_n^*) g_{2,n}^*(z) e^{-\omega_n^* z} \right] \tag{7}$$

Note that when using equations 6-7, the assumption is that the coupling κ is zero for $z \leq 0$. In addition, the specified response will theoretically only appear at $z = \infty$. Fortunately, κ usually converges towards zero at an earlier stage. Still, depending on how early we decide to truncate our coupler, the coupler's response will always differ somewhat from the specifications.

A number of different polynomial approximations may be used, for the present we only provide results of a Butterworth response of order n defined by:

$$|S(\delta)|^2 = \frac{b^2}{1 + \left(\frac{\delta}{\delta_c}\right)^{2n}} \tag{8}$$

Here b has a value less than one, and we set the scaling factor $\delta_c = 1$ in the following discussions. From equation 8 we may deduce:

$$\begin{aligned} |R(\delta)|^2 &= 1 - \frac{b^2}{1 + \delta^{2n}} \\ &= \frac{\delta^{2n} + (1 - b^2)}{1 + \delta^{2n}} \end{aligned} \tag{9}$$

$$|r|^2 = \frac{b^2}{(1-b^2)+\delta^{2n}} \quad (10)$$

For $G(u)$ in equation 4 we then get:

$$\begin{aligned} |G(u)|^2 &= G(u)G^*(u) = -G(u)G(-u) \\ &= \frac{b^2}{(1-b^2)+(-u^2)^n} \quad (11) \end{aligned}$$

The $2n$ poles of $|G(u)|$ may now easily be found [5]:

$$\rho_k = \begin{cases} (1-b^2)^{\frac{1}{2n}} e^{j\frac{(2k-1)\pi}{2n}} & n \text{ even} \\ (1-b^2)^{\frac{1}{2n}} e^{j\frac{k\pi}{n}} & n \text{ odd} \end{cases} \quad (12)$$

where

$$\text{for } n \text{ even}, k = 1, 2, \ldots, 2n \quad (13)$$
$$\text{and for } n \text{ odd}, k = 0, 1, \ldots, 2n-1 \quad (14)$$

The poles in the left half plane are identified with $G(u)$, whereas the other poles are identified with $G(-u)$. Note that $b = 1$ means that all the poles are at the origin. In order to have distinct poles, which is one of the requirements for the GLM technique, b needs to be less than one. In practice this is, however, not a problem since b without any numerical difficulty can be set much closer to one than any physical coupler can achieve. For the third order filter , $n = 3$, we thus obtain the following relations [3]:

$$\begin{aligned} P(u) &= jb \quad (15) \\ Q(u) &= \prod_{k=2}^{4} \left[-u + \left(1-b^2\right)^{\frac{1}{6}} e^{j\frac{\pi k}{3}}\right] \quad (16) \\ F(u) &= 1 - u^6 \quad (17) \end{aligned}$$

The roots of $Q = 0$ and $F = 0$ may now be easily found from equations 16 and 17, respectively, and we next solve equation set 6. Using equation 7 the coupling coefficient κ may be calculated. In order to set the coupling and interaction lengths in perspective, it may help to compare the results with a uniform

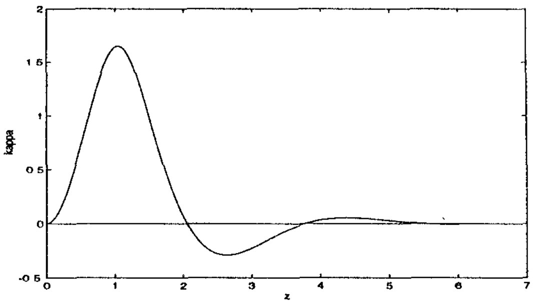

Figure 2: The spatially varying coupling function κ of a third order Butterworth filter function with cutoff at $\delta = 1$ and with $b^2 = 0.99$ calculated using the GLM technique.

directional coupler modulator that is one coupling length long. If the "switching δ" is unity, the coupling length will to be $\sqrt{3}\pi/2 \approx 2.7$, and the coupling coefficient κ will be $1/\sqrt{3} \approx 0.58$. Thus the maximum value of the coupling coefficient is three times as large in the Butterworth function. The coupling length is in this case harder to calculate, since the device ideally should be infinitely long. A truncation at $z = 6$ does, however, give satisfactory results, as shown in figure 3. Note that for each zero in the spatially varying coupling function we get a sign shift. This is a problem we encountered in most of the coupling function we have synthesized. A "negative" coupling is in itself meaningless, it may be seen that a sign shift in the coupling corresponds to a phase shift between the two modes. There is

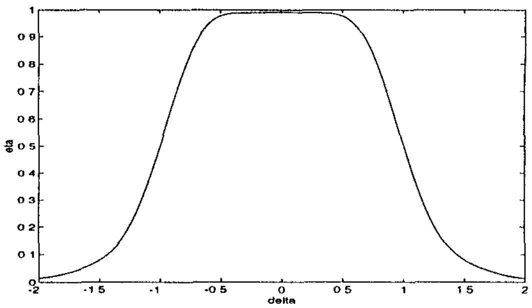

Figure 3: Third order Butterworth filter function. The response η is reconstructed from the spatially varying filter function shown in figure 2. The coupling function was truncated at $z = 6$.

no way of avoiding these sign shifts. Neither a truncation at the first zero, use of only the absolute value, nor avoiding the zeros by adding a constant to κ give even close to satisfactory results. A sign change of the coupling function corresponds to a 180 degree phase shift, and this is easily achieved using an extra half wavelength of line, which is very small, in one of the arms of the coupler. Thus, the coupler from the truncated Butterworth coupling function will have three regions where the coupling rises to the appropriate maximum, and at every sign change the coupling goes to zero, where the extra length of line is added in one of the arms. In summary we have shown that the response function of a coupler modulator may be synthesized by obtaining the coupling function. The realization of the coupler with the specified coupling function is then the next step.

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